Pacific Science Review B: Humanities and Social Sciences 1 (2015) 8–14

HOSTED BY

ELSEVIER

Contents lists available at ScienceDirect

Pacific Science Review B: Humanities and Social Sciences

journal homepage: www.journals.elsevier.com/pacific-science-review-b-humanities-and-social-sciences/

A joint venturing of single supplier and single retailer under variable price, promotional effort and service level

Arpita Roy [a], Shib Sankar Sana [b,*], Kripasindhu Chaudhuri [c]

[a] Department of Mathematics, Heritage Institute of Technology, Kolkata 700107, India
[b] Department of Mathematics, Bhangar Mahavidyalaya, Bhangar, Kolkata 743502, 24PGS (South), India
[c] Department of Mathematics, Jadavpur University, Kolkata 700032, India

ARTICLE INFO

Article history:
Available online 27 November 2015

Keywords and phrases:
Two-echelon supply chain
Pricing
Newsvendor problem
Uncertain demand

ABSTRACT

This paper analyses a two-echelon supply chain composed of one supplier and one retailer. The market demand is assumed to be uncertain and considered to be retail price dependent and dependent on the supplier's service level and on the retailer's promotional effort. The unsold items at the retailer are repurchased by the supplier at a price less than the sales prices. Conversely, the retailer encounters shortages because the demand is naturally uncertain. The optimal order quantity, selling price, promotional effort and service level are evaluated analytically as well as numerically for single period newsvendor-type demand patterns. The profit functions of the supplier and the retailer are analysed and compared in accordance with Stakelberg and integrated approaches. Computational results show that an integrated system is always beneficial for the members of the chain.

1. Introduction

The objective of supply chain management is to generally target performance enhancement by either reducing costs or escalating profits. There are vast methods of doing either; therefore, the most effective means of profit enhancement is boosting market demand by various means. Given today's furiously competitive market scenario, the foremost objective of an efficient supply chain manager is to consider all of the issues pertaining to demand enhancement. The concerned management must address the issues that affect the market demand and outline the strategies of the chain accordingly. The very first step is to identify the most sensitive factors that improvise market demand. Of all such factors, the product's retail price is the most important factor that affects demand. Usually the demand decreases with the retail price increment. In addition to the retail price, the market demand depends on certain non-price factors such as the quality of the product, the brand value, and the availability in the market. There are few factors other than these non-price factors that positively improve the product's demand. For example, different types of promotional activities adopted by supply chain members are to enhance the demand of the commodity. Such activities include advertisements, free gifts, delays in payment, and discount offers. If a huge number of consumers become knowledgeable regarding the salient attractive features and usefulness of the product through promotional activities, the demand for the product will improve. Another fruitful means to enhance demand is furnishing after-sales service. Bestowing before and after-sale servicing such as bestowing free servicing and repairing through a stipulated time period or facilitating replacement of the product if damaged or underperforming are generally very helpful to capturing a wide range of customers. Very often, providing a high service level or investing greatly in promotional activities is very costly and may cause high expenditure levels. Therefore, chain members must invest in an optimal desired effort level to increase potential customers. In both inventory and supply chain literature, researchers have determined sales effort as a factor that increases demand in a single retailer scenario as well as a tool that benefits the retailer in capturing more of the total market demand.

In recent decades, there has been extensive research regarding different forms of demands having price dependency; for example, Wee (1997), Qin et al. (2007) and Ghosh et al. (2011) considered linear price dependent demand, and Ouyang et al. (2008) and Ziaee

* Corresponding author.
E-mail address: shib_sankar@yahoo.com (S.S. Sana).
Peer review under responsibility of Far Eastern Federal University, Kangnam University, Dalian University of Technology, Kokushikan University.

http://dx.doi.org/10.1016/j.psrb.2015.11.001

and Bouquard (2010) assumed the iso-elastic form of price dependency. Huang and Li (2001), Huang et al. (2002) and Li et al. (2002) studied the role of vertical co-op advertising efficiency with respect to dealings between a manufacturer and a retailer through brand names. Bernstein and Federgruen (2007) examined coordination mechanisms for supply chains under price and service competition. Lau and Lau (2003) explicitly studied the effect of different demand patterns on the optimal decisions of a multi-echelon supply chain. Panda (2011) addressed an optimal pricing and replenishment policy in a declining price sensitive environment under continuous unit cost decreases. Panda et al. (2012) studied the problem of optimal pricing and economic order quantity in a stock and price sensitive demand environment. Javid and Hoseinpour (2011) investigated the coordination of co-op advertising decisions in a supply chain with one manufacturer and one retailer. Wang et al. (2011) coordinated co-op advertising models with one manufacturer and two competitive retailers. Recently, Javid and Hoseinpour (2012) re-addressed the static, single-period co-op advertising model that was introduced and studied earlier by Huang and Li (2001) and subsequently by several other authors using game theory.

Despite the fact that sales efforts are utilised in the real world to promote huge sales, the quantity of effort and money expended on providing after-sale service and the impact of this usually differ from product to product. It is generally observed that the effort level usually varies with the price of the product. If the selling price is higher, higher sales effort and facilities offered after sales associated with the product are needed. Therefore, the resulting effect of providing sales efforts to a particular product on the profits earned from selling the product by channel partners must be examined before deciding whether to provide such a facility to end customers. The effect also depends on the company's reliability and brand value; these play vital roles to control and determine the effect of such efforts on demand. Remembering the previously noted facts, a significant quantity of research has been performed in recent years on how the sales effort affects the market demand; the research also includes how to exert such effort efficiently. Jorgensen et al. (2000, 2001, 2003) and Jorgensen and Zaccour (2003) extensively studied the long-term multi-period co-op advertising relation between a manufacturer and a retailer through dynamic models. Krishnan et al. (2004) showed that buy-backs with promotional cost-sharing agreements can be coordinated. Service as well as promotion thus affect both consumer demand and the retailer's sales motivation. Several authors have established that certain main results and conclusions in the supply chain analysis may be affected by the choice of the demand model; in addition, the results are sensitive (Jeuland and Shugan, 1988; Granot and Yin, 2007). Roy and Chaudhuri (2010) analysed a model of perishable items with time and price dependent demand considering the time value of money. Recently, the EOQ (economic order quantity) and the EPQ (economic production quantity) model for price and promotional effort sensitive demand have been studied extensively by Taleizadeh et al. (2015a, 2015b), Cardenas-Barron and Sana (2014, 2015) and Cardenas-Barron et al. (2014). Modak et al. (2015) introduced corporate social responsibility in a two-echelon dual-channel supply chain model in which the coordination of all unit quantity discounts using a franchise fee agreement and dividing the surplus profit through bargaining are discussed analytically.

This paper studies the effect of promotional effort and after-sales service level on demand uncertainty, within a two-echelon single-period NV (newsvendor) environment using the pricing strategies of a profit maximizing supplier/retailer. According to the author's best knowledge, no one has yet used promotional efforts coupled with before/after-sales service efforts as a tool for enhancing the demand while simultaneously using buy-back contracts for supply chain coordination. Most cited papers solely consider the effect of price. It is also observed that the demand is not only dependent on the product's price but also on the manufacturer's effort. To develop the market, the supplier/agent applies selling efforts for the product such as stipulation of the product information in the market, advertisements, expenditures on social activities, greening efforts, free repair or after-sales service; these are very common in practice. Coexistence of the above factors is common in most retail sectors as strong efforts to improve market demand. From the above literature, the consequence of promotional effort on supply chain coordination as well as the influence of supplier's extra marketing efforts are ignored, particularly in the NV environment area in which the demand is stochastic in nature. The literature on the NV is very voluminous and well known in terms of applicability. Excellent summaries on the subject appear in Petruzzi and Dada (1999), Silver et al. (1998), Khouja (1999), Chan et al. (2004), and Lau and Lau (2001). A primary difficulty in this type of research is the problem of designing contracts that will be rendered beneficial to the various members of the chain. The main objective of the classical newsboy problem is to maximize the expected profit. Many other approaches have also been included in the classical newsboy problem, for example, the mathematical programming approach was used to find the solution. Wang (2010) studied a game theoretic approach including multiple newsvendors with loss aversion preferences that were competing for inventory from a risk-neutral supplier. Ozler et al. (2009) proposed the multi-product newsboy problem under a Value at Risk (VaR) constraint. Soni and Shah (2011) studied an EPQ model for deterministic, stochastic or fuzzy demand to minimize the cost function. Cardenas-Barron et al. (2012) proposed a heuristic algorithm to determine an optimal solution of the vendor management inventory system with a multi-product and a multi-constraint based on EOQ with backorders considering two classical backorders costs: linear and fixed. In newsvendor modelling, the noteworthy works of Lau and Lau (1988), Wu et al. (2007), Taleizadeh et al. (2012), and Sana (2011, 2012) should be noted.

In our model, we develop a two-layer supply chain consisting of one supplier and one retailer. The demand of the end customers is stochastic because the arrival of the customers is not certain in practice. The demand function here has been considered as a function depending on three prime factors: price, promotional effort as well as sales service efforts. We have considered the linear demand function case. Because the demand is uncertain, the retailer may overstock or understock the products. Because of overstocking, the unsold items are returned to the supplier at a lower price than the supplier's wholesale price, whereas the retailers must bear the penalty cost for understocking. We have developed the profit functions for the supplier and the retailer while maintaining the price-promotional effort and sales service effort-dependent demand in the account. These supplier and retailer profits are formulated and optimized by utilizing Stakelberg and integrated approaches. We have also conducted a comparative study among the above approaches, which ultimately suggests that the integrated system provides the best profit.

The remainder of this paper is organized as follows: Section 2 explains the fundamental notations and assumptions, Section 3 provides mathematical formulations and an analysis of the model, and in Section 4, we discuss numerical examples. Section 5 presents a conclusion regarding this paper's findings.

2. Fundamental assumptions and notation

The following assumptions are made to develop the model:

Assumption

(i) The model is developed for a single item.
(ii) The model is associated with a two-echelon supply chain composed of one supplier and one retailer.
(iii) The Demand rate of the members of the chain is assumed to be uncertain and price, promotional effort and service level sensitive.
(iv) The chain uses a buy-back policy.
(v) The lead time is negligible.
(vi) Shortages at the retailers are permitted.
(vii) The replenishment rate is instantaneously infinite; however, its size is finite.

Notation

Q – Retailer's Order Quantity.
x – A part of demand quantity (units/month) during a period, which is a random variable that follows a probability distribution.
$f(x)$ – Probability density distribution function of x.
$F(x)$ – Cumulative distribution function of x.
$F^{-1}(x)$ – Inverse function of F.
p – Unit retail price (\$/unit) for retailer.
$D(p,s,u)$ – Demand (units/month), which is a function of the retail price and the service level. This is provided by the supplier and the advertising expenditure of the retailer.
s – Service level bore by the supplier for the end customer.
w – Selling price (\$/unit) per unit of the supplier to the retailer (unit purchasing cost of products at the retailer).
v_1 – Unit salvage value/return price (\$/unit) under a buy-back contract of unsold goods of the retailer provided by the supplier.
v_2 – Unit salvage value/return price (\$/unit) of the returned items of the retailer at the supplier by selling to others.
c – Purchasing cost/production cost (\$/unit) of the supplier.
r – Shortage cost (\$/unit) of the retailer.
u – Retailer's promotional effort.
β – Denotes the demand sensitivity of the retailer on its own sales price p.
γ – Denotes the demand sensitivity of the retailer on the supplier's service level.
δ – Denotes the demand sensitivity of the retailer on its promotional effort.
μ – Denotes the mean demand of the retailer.
t_1 – Denotes the fraction of expenditure incurred for service facilities shared by the supplier.
t_2 – Denotes the fraction of expenditure incurred for the promotional effort shared by the supplier.
$E(S)$ – Expected profit (\$/month) function of the supplier.
$E(R)$ – Expected profit (\$/month) function of the retailer.
EIP – Expected integrated profit (\$/month) of the chain.
x^+ – $Max[x,0]$, the positive part of x.

3. Mathematical formulation and analysis of the model

We consider a two-echelon supply chain consisting of a single supplier and single retailer. The supplier supplies solely to a single downstream retailer. The end consumers' perception of value and their purchase decisions are not only influenced exclusively by the item's selling price but also by the service level and promotional efforts that accompany it. Here, service is deemed to broadly represent all forms of the demand-enhancing effort, including customer service before and after the sale, in-store promotions, and advertising and warranty offerings. Many such services can be provided either by the supplier, or they can be delegated to the dealer. The supplier and the manufacturer share the cost invested in service facilities and promotional efforts. In a decentralised system, the supplier that can observe the retailer's decisions possesses an advantageous position; however, he cannot control what the retailer chooses to do. The retailer makes his/her own decisions independently. Similarly, the supplier may decide on the service level, the order quantity and the promotional effort independently, and the retailer then makes the optimal decision regarding the selling price of the products. However, in an integrated/collaborating system, the supplier and the retailer make decisions jointly.

In this paper, the demand is assumed to be the function of retail price p and promotional effort u provided by the retailer and the service level s of the supplier. The functional form of demand is

$$D(p,s,u) = x - \beta p + \gamma s + \delta \tag{1}$$

Here, the demand function is decreasing in its own retail price, increasing in the service level of the supplier and increasing in the promotional effort u. In this section, we assume that the supplier declares beforehand that he/she would share a fraction (t_1, t_2) of the expenditures for services and promotional effort, respectively. The parameters $\beta(\geq 0)$, $(\gamma \geq 0)$ and $\delta(\geq 0)$ measure the responsiveness of market demand to its own price and service level and promotional activity, respectively. The random variable x describes the base-case potential market size for the product that follows p.d.f. $f(x)$, i.e., $\int_0^\infty f(x)dx = 1$. The expected profit functions of the supplier and the retailer are as follows.

$$E1[S(Q,p,s,u)] = (w-c)Q - (v_1 - v_2)E(A-x)^+ - \frac{1}{2}t_1 ms^2 - t_2 nu^2 \tag{2}$$

and

$$E2[R(Q,p,s,u)] = p(\mu - \beta p + \gamma s) - wQ + v_1 E(A-x)^+ - rE(x-A)^+ - \frac{1}{2}(1-t_1)ms^2 - (1-t_2)nu^2, \tag{3}$$

where

$$A = (Q + \beta p - \gamma s - \delta u), \tag{4}$$

$$E(A-x)^+ = \int_0^A (A-x)f(x)dx, \tag{5}$$

$$E(x-A)^+ = \int_A^\infty (x-A)f(x)dx \tag{6}$$

and

$$F(A) = \int_0^A f(x)dx. \tag{7}$$

Now, our main objective is to determine the optimal values of the EOQ, the sales prices, the service level and the promotional effort such that the chain's profit is maximized. The main objective of our model is to maximize the profits of the chain as well as the profits of the individual members of the chain. We shall study the decision mechanism under the following scenarios:

Case-*I*: Centralized Supply Chain (CS)

In the centralized model of this system, both the supplier and the retailer make decisions after consulting each other. Then, the prime objective of the members of the chain is to maximize the integrated expected profit of the system. Therefore, the expected profit of the chain, combining Eqs. (2) and (3), is

$$EIP = E1 + E2 = p(\mu - \beta p + \gamma s) - cQ + v_2 E(A - x)^+ - rE(x - A)^+ - 1/2ms^2 - nu^2 \tag{8}$$

Now, differentiating *EIP* with respect to Q,p,s and u, we have

$$\frac{\partial EIP}{\partial Q} = (r - c) - (r - v_2)F(A), \tag{9}$$

$$\frac{\partial EIP}{\partial p} = (\mu - 2\beta p + \gamma s + \delta u + \beta r) - (r - v_2)\beta F(A), \tag{10}$$

$$\frac{\partial EIP}{\partial s} = (\gamma p - ms - r\gamma) + (r - v_2)\gamma F(A), \tag{11}$$

$$\frac{\partial EIP}{\partial u} = (\delta p - 2nu - r\delta) + (r - v_2)\delta F(A), \tag{12}$$

$$\frac{\partial^2 EIP}{\partial Q^2} = -(r - v_2)f(A) < 0;\ \forall\, r > v_2, \tag{13}$$

$$\frac{\partial^2 EIP}{\partial p^2} = -2\beta - (r - v_2)\beta^2 f(A) < 0;\ \forall\, r > v_2, \tag{14}$$

$$\frac{\partial^2 EIP}{\partial s^2} = -m - (r - v_2)\gamma^2 f(A) < 0;\ \forall\, r > v_2, \tag{15}$$

$$\frac{\partial^2 EIP}{\partial u^2} = -2n - (r - v_2)\delta^2 f(A) < 0; \forall\, r > v_2, \tag{16}$$

$$H = \begin{pmatrix} -(r-v_2)f(A) & -(r-v_2)\beta f(A) & (r-v_2)\gamma f(A) & (r-v_2)\delta f(A) \\ -(r-v_2)\beta f(A) & -2\beta-(r-v_2)\beta^2 f(A) & \gamma+(r-v_2)\beta\gamma f(A) & \delta+(r-v_2)\beta\delta f(A) \\ (r-v_2)\gamma f(A) & \gamma+(r-v_2)\beta\gamma f(A) & -m-(r-v_2)\gamma^2 f(A) & -(r-v_2)\gamma\delta f(A) \\ (r-v_2)\delta f(A) & \delta+(r-v_2)\beta\delta f(A) & -(r-v_2)\gamma\delta f(A) & -2n-(r-v_2)\delta^2 f(A) \end{pmatrix}$$

$$\frac{\partial^2 EIP}{\partial Q \partial p} = \frac{\partial^2 EIP}{\partial p \partial Q} = -(r - v_2)\beta f(A) < 0; \forall\, r > v_2, \tag{17}$$

$$\frac{\partial^2 EIP}{\partial Q \partial s} = \frac{\partial^2 EIP}{\partial s \partial Q} = (r - v_2)\gamma f(A) > 0;\ \forall\, r > v_2, \tag{18}$$

$$\frac{\partial^2 EIP}{\partial Q \partial u} = \frac{\partial^2 EIP}{\partial u \partial Q} = (r - v_2)\delta f(A) > 0;\ \forall\, r > v_2, \tag{19}$$

$$\frac{\partial^2 EIP}{\partial p \partial s} = \frac{\partial^2 EIP}{\partial s \partial p} = \gamma + (r - v_2)\beta\gamma f(A) > 0;\ \forall\, r > v_2, \tag{20}$$

$$\frac{\partial^2 EIP}{\partial p \partial u} = \frac{\partial^2 EIP}{\partial u \partial p} = \delta + (r - v_2)\beta\delta f(A) > 0; \forall\, r > v_2, \tag{21}$$

$$\frac{\partial^2 EIP}{\partial s \partial u} = \frac{\partial^2 EIP}{\partial u \partial s} = -(r - v_2)\gamma\delta f(A) < 0; \forall\, r > v_2. \tag{22}$$

For the maximum value of *EIP*, Eqs. (9)–(12) are individually zero. Then, solving Eqs. (9)–(12), we have the stationary points as follows

$$u^* = \frac{m\delta(\mu - c\beta)}{(4\beta mn - 2n\gamma^2 - \gamma\delta m)}, \tag{23}$$

$$s^* = \frac{2n\gamma u}{m\delta}, \tag{24}$$

$$p^* = \frac{1}{\delta}(2nu + c\delta), \tag{25}$$

$$Q^* = -\beta p^* + \gamma s^* + \delta u^* + F^{-1}\left(\frac{r - c}{r - v_2}\right). \tag{26}$$

The associated Hessian matrix is given by

$$H = \begin{pmatrix} \frac{\partial^2 EIP}{\partial Q^2} & \frac{\partial^2 EIP}{\partial Q \partial p} & \frac{\partial^2 EIP}{\partial Q \partial s} & \frac{\partial^2 EIP}{\partial Q \partial u} \\ \frac{\partial^2 EIP}{\partial Q \partial p} & \frac{\partial^2 EIP}{\partial p^2} & \frac{\partial^2 EIP}{\partial p \partial s} & \frac{\partial^2 EIP}{\partial p \partial u} \\ \frac{\partial^2 EIP}{\partial Q \partial s} & \frac{\partial^2 EIP}{\partial p \partial s} & \frac{\partial^2 EIP}{\partial s^2} & \frac{\partial^2 EIP}{\partial s \partial u} \\ \frac{\partial^2 EIP}{\partial Q \partial u} & \frac{\partial^2 EIP}{\partial p \partial u} & \frac{\partial^2 EIP}{\partial s \partial u} & \frac{\partial^2 EIP}{\partial u^2} \end{pmatrix}$$

Substituting the above second-order partial derivatives, the Hessian matrix is

To check the nature of the Hessian matrix, we must show that the matrix must be definitely negative for concavity of the *EIP*, i.e., the eigenvalues of the matrix are all negative real numbers.

Case-*II*: Partial Decentralized Supply Chain (DCS)

In this case, the supplier determines the optimal values of Q,s and u, and the retailer determines the optimal value of p. Now, differentiating $E1$ partially with respect to Q,s,u, and $E2$ with respect to p, we have

$$\frac{\partial E1}{\partial Q} = (w - c) - (v_1 - v_2)F(A), \tag{27}$$

$$\frac{\partial E1}{\partial s} = -mt_1 s + (v_1 - v_2)\gamma F(A), \tag{28}$$

$$\frac{\partial E1}{\partial u} = -2nt_2 u + (v_1 - v_2)\delta F(A), \tag{29}$$

$$\frac{\partial E2}{\partial p} = (\mu - 2\beta p + \gamma s + \delta u + \beta r) - (r - v_1)\beta F(A). \tag{30}$$

Equating Eqs. (27)–(30) to zero and solving these, we have the optimal values of (Q,p,s,u) as follows:

$$u^* = \frac{\delta(w-c)}{2nt_2}, \tag{31}$$

$$s^* = \frac{\gamma(w-c)}{mt_1}, \tag{32}$$

$$p^* = \frac{1}{2\beta}\left\{\mu + \frac{\gamma^2(w-c)}{mt_1} + \frac{\delta^2(w-c)}{2nt_2} + r\beta - \frac{(w-c)(r-v_1)\beta}{v_1 - v_2}\right\}, \tag{33}$$

$$Q^* = -\beta p^* + \gamma s^* + \delta u^* + F^{-1}\left(\frac{w-c}{v_1 - v_2}\right). \tag{34}$$

Here, the Hessian matrix of $E1$ is

$$H = \begin{pmatrix} -(v_1-v_2)f(A) & (v_1-v_2)\gamma f(A) & (v_1-v_2)\delta f(A) \\ (v_1-v_2)\delta f(A) & -mt_1 - (v_1-v_2)\gamma^2 f(A) & -(v_1-v_2)\gamma\delta f(A) \\ (v_1-v_2)\delta f(A) & -(v_1-v_2)\gamma\delta f(A) & -2nt_2 - (v_1-v_2)\delta^2 f(A) \end{pmatrix}$$

and the second derivative of $E2$ with respect to p is

$$\frac{\partial^2 E2}{\partial p^2} = -2\beta - (r - v_1)\beta^2 f(A) < 0; \forall\, r > v_1. \tag{35}$$

Here, the profit function E1 would be concave if the Hessian matrix H at the above stationery point is definitely negative, i.e., the eigenvalues of the matrix are all negative real numbers.

Case-III: DCS when the Retailer is the decision maker

In this case, the supplier is the follower of the decisions made by the retailer. Then, the retailers optimize their respective lot sizes and sales prices, service levels and promotional efforts, remembering their own profit. Then, the partial derivatives of $E2$ are

$$\frac{\partial E2}{\partial Q} = (r - w) - (r - v_1)F(A), \tag{36}$$

$$\frac{\partial E2}{\partial p} = (\mu - 2\beta p + \gamma s + \delta u + \beta r) - (r - v_1)\beta F(A), \tag{37}$$

$$\frac{\partial E2}{\partial s} = (\gamma p - m(1 - t_1)s - r\gamma) + (r - v_1)\gamma F(A), \tag{38}$$

$$\frac{\partial E2}{\partial u} = (\delta p - 2n(1 - t_2)u - r\delta) + (r - v_1)\delta F(A), \tag{39}$$

$$\frac{\partial^2 E2}{\partial Q^2} = -(r - v_1)f(A) < 0; \forall\, r > v_1, \tag{40}$$

$$\frac{\partial^2 E2}{\partial p^2} = -2\beta - (r - v_1)\beta^2 f(A) < 0; \forall\, r > v_1, \tag{41}$$

$$\frac{\partial^2 E2}{\partial s^2} = -m(1 - t_1) - (r - v_1)\gamma^2 f(A) < 0; \forall\, r > v_1, \tag{42}$$

$$\frac{\partial^2 E2}{\partial u^2} = -2n(1 - t_2) - (r - v_1)\delta^2 f(A) < 0; \forall\, r > v_1, \tag{43}$$

$$\frac{\partial^2 E2}{\partial Q \partial p} = \frac{\partial^2 EIP}{\partial p \partial Q} = -(r - v_1)\beta f(A) < 0; \forall\, r > v_1, \tag{44}$$

$$\frac{\partial^2 E2}{\partial Q \partial s} = \frac{\partial^2 EIP}{\partial s \partial Q} = (r - v_1)\gamma f(A) > 0; \forall\, r > v_1, \tag{45}$$

$$\frac{\partial^2 E2}{\partial Q \partial u} = \frac{\partial^2 EIP}{\partial u \partial Q} = (r - v_1)\delta f(A) > 0; \forall\, r > v_1, \tag{46}$$

$$\frac{\partial^2 E2}{\partial p \partial s} = \frac{\partial^2 EIP}{\partial s \partial p} = \gamma + (r - v_1)\beta\gamma f(A) > 0;\ \forall r > v_1, \tag{47}$$

$$\frac{\partial^2 E2}{\partial p \partial u} = \frac{\partial^2 EIP}{\partial u \partial p} = \delta + (r - v_1)\beta\delta f(A) > 0; \forall\, r > v_1, \tag{48}$$

$$\frac{\partial^2 E2}{\partial s \partial u} = \frac{\partial^2 EIP}{\partial u \partial s} = -(r - v_1)\gamma\delta f(A) < 0; \forall\, r > v_1. \tag{49}$$

Equating Eqs. (36)–(39) to zero and solving these, we have the optimal solution as follows:

$$u^* = \frac{m\delta(\mu - w\beta)}{\left(4\beta mn(1 - t_2) - \delta^2 m - \frac{2n\gamma^2}{(1-t_1)}\right)}, \tag{50}$$

$$s^* = \frac{2n\gamma(1 - t_2)u^*}{m\delta(1 - t_1)}, \tag{51}$$

$$p^* = \frac{1}{\delta}\left\{r\delta + 2n(1 - t_2)u^* - (r - w)\delta\right\}, \tag{52}$$

$$Q^* = -\beta p^* + \gamma s^* + \delta u^* + F^{-1}\left(\frac{r-w}{r-v_1}\right). \tag{53}$$

Now, the Hessian matrix of $E2$ is

$$H = \begin{pmatrix} -(r-v_1)f(A) & -(r-v_1)\beta f(A) & (r-v_1)\gamma f(A) & (r-v_1)\delta f(A) \\ -(r-v_1)\beta f(A) & -2\beta - (r-v_1)\beta^2 f(A) & \gamma + (r-v_1)\beta\gamma f(A) & \delta + (r-v_1)\beta\delta f(A) \\ (r-v_1)\gamma f(A) & \gamma + (r-v_1)\beta\gamma f(A) & -m(1-t_1) - (r-v_1)\gamma^2 f(A) & -(r-v_2)\gamma\delta f(A) \\ (r-v_1)\delta f(A) & \delta + (r-v_1)\beta\delta f(A) & -(r-v_1)\gamma\delta f(A) & -2n(1-t_2) - (r-v_1)\delta^2 f(A) \end{pmatrix}$$

The above optimal value of (u^*, s^*, p^*, Q^*) provides maximum profit of $E2$ if the above Hessian matrix is definitely negative, i.e., the eigenvalues of the matrix at the optimal values of the decision variables are all negative real numbers.

Case-*IV:* Profit sharing in the integrated system

In this situation, the integrated profit EIP is distributed according to the investments of the channel members in the business. Then, the required profit of the supplier and the retailer are as follows:

$$E1^{**} = EIP^*\left(\frac{\text{Invested cost of the supplier}}{\text{Total cost of the whole system}}\right) \tag{54}$$

and

$$E2^{**} = EIP^*\left(\frac{\text{Invested cost of the retailer}}{\text{Total cost of the whole system}}\right) \tag{55}$$

The profit function EIP is concave at the optimal values of the decision variables because the eigenvalues (−2.11374, −0.456307, −0.387495, −0.170715) are all negative real numbers.

4. Numerical example

We consider the distribution function of x as follows: $\{f(x) = a_0 + a_1 x + a_2 x^2; \forall 0 \le x \le M\}$, where $a_1 = \frac{3}{M^3}(10\mu^2 + 10\sigma^2 + 3M^2 - 12\mu M)$, $a_2 = -\frac{12}{M^4}(15\mu^2 + 15\sigma^2 + 3M^2 - 16\mu M)$, $a_3 = \frac{30}{M^5}(6\mu^2 + 6\sigma^2 + M^2 - 6\mu M)$ and μ and σ are the mean and the standard deviation of $f(x)$, respectively. The values of the other key parameters are as follows: $M = 100$, $\mu = 50$, $\sigma = 10$, $\beta = 0.70$, $\gamma = 0.45$, $\delta = 0.35$, $w = 50$, $v_1 = 40$, $v_2 = 30$, $r = 55$, $c = 35$, $m = 0.45$, $n = 0.23$, $t_1 = 1/2$ and $t_2 = 1/4$. Then, the optimal solutions for different cases are shown in Table 1. In Table 1, it is observed that the integrated system is the best strategy for the members of the chain; the integrated profit is distributed according to their investment in the business. In the integrated system, the optimal solution is ($u^* = 28.36$, $s^* = 37.30$, $p^* = 72.30$, $Q^* = 41.10$) and the costs of the supplier and the retailer invested in the joint venture are 2253.30 and 2374.25, respectively. Now, the expected profits of the supplier and the retailer are 262.20 and 275.66, respectively.

Table 1
Optimal solution in different cases.

Scenarios	Optimal values of variables						
	u^*($)	s^*($)	p^*($)	Q^*(units)	$E1^*$($)	$E2^*$($)	EIP^*($)
Case-I	28.38	37.30	72.30	41.10	262.20	275.66	537.86
Case-II	45.65	30.00	73.02	68.25	406.68	−197.99	208.69
Case-III	105.00	207.00	153.50	64.40	−4511.16	701.86	−3809.30

5. Conclusion

We have studied the management problems related to an economic order quantity, a promotional effort and a service level in a supply chain consisting of one supplier and one retailer, assuming uncertainty in the market demand. Managing uncertainty has become increasingly challenging over time. Here, the demand of the customers is influenced by both price and advertising expenditures as well as sales service efforts. In the proposed model, the relation between price and demand has a relatively general form compared with the classic linear relation. We have also considered centralized and decentralized systems. Ultimately, comparative study among all of the different approaches has been performed, and the results reflect the significant effect in the demand-price function and may provide a set of optimal values of decision variables and supply chain members' profit. The practical aspects of our proposed model include addressing the advertising and sales service saturation effect in demand function modelling and different channel structures (cooperation and the case of a dominant member). The results show that the profit of the supply chain achieves its highest value when the chain members cooperate, i.e., they conduct business in collaboration.

The proposed model can be extended immediately considering supply disruption because of transportation problems and the availability of the products in the market. Conversely, we have developed the model for continuous variables, which is the main limitation of our model. This limitation can be waived by considering the proposed model for discrete types of decision variables. Moreover, this research can be extended directly by considering a three-level supply chain (a retailer, a manufacturer and a supplier) or multi-supplier situations instead of one or more uncertainties such as the selling price and the supplier's lead time uncertainties. Another possibility is to study the model by evaluating other forms of contracts such as a revenue sharing contract or any other new flexibility contract instead of using a buy-back contract.

References

Bernstein, F., Federgruen, A., 2007. Coordination mechanisms for supply chains under price and service competition. Manuf. Serv. Oper. Manag. 9 (3), 242–262.
Cardenas-Barron, L.E., Trevino-Garza, G., Wee, H.M., 2012. A simple and better algorithm to solve the vendor managed inventory control system of multi-product multi-constraint economic order quantity model. Expert Syst. Appl. 39 (2), 3888–3895.
Cardenas-Barron, L.E., Chung, K.J., Trevio-Garza, G., 2014. Celebrating a century of the economic order quantity model in honor of Ford Whitman Harris. Int. J. Prod. Econ. 155, 1–7.
Cardenas-Barron, L.E., Sana, S.S., 2014. A production-inventory model for a two-echelon supply chain when demand is dependent on sales teams' initiatives. Int. J. Prod. Econ. 155, 249–258.
Cardenas-Barron, L.E., Sana, S.S., 2015. Multi-item EOQ inventory model in a two-layer supply chain while demand varies with a promotional effort. Appl. Math. Model. 39 (21), 6725–6737.

Chan, L.M.A., Shen, Z.J.M., Swan, D., Simchi-Levi, J.L., 2004. Coordination of pricing and inventory decisions: a survey and classification. In: Simchi-Levi, D., Wu, S.D., Shen, Z.J.M. (Eds.), Handbook of Quantitative Supply Chain Analysis: Modeling in the E-business Era. Kluwer, Boston, MA, pp. 335–392.

Ghosh, S.K., Khanra, S., Chaudhuri, K.S., 2011. Optimal price and lot size determination for a perishable product under conditions of finite production, partial backordering and lost sale. Appl. Math. Comput. 217 (13), 6047–6053.

Granot, D., Yin, S., 2007. On sequential commitment in the price-dependent news vendor model. Eur. J. Oper. Res. 177 (2), 939–968.

Huang, Z., Li, S.X., 2001. Co-op advertising models in manufacturer-retailer supply chains: a game theory approach. Eur. J. Oper. Res. 135 (3), 527–544.

Huang, Z., Li, S.X., Mahajan, V., 2002. An analysis of manufacturer-retailer supply chain coordination in cooperative advertising. Decis. Sci. 33 (3), 469–494.

Javid, A.A., Hoseinpour, P., 2011. A game-theoretic analysis for coordinating cooperative advertising in a supply chain. J. Optim. Theory Appl. 149 (1), 138–150.

Javid, A.A., Hoseinpour, P., 2012. On a cooperative advertising model for a supply chain with one manufacturer and one retailer. Eur. J. Oper. Res. 219 (2), 458–466.

Jeuland, A.P., Shugan, S.M., 1988. Channel of distribution profits when channel members form conjectures. Mark. Sci. 7 (2), 202–210.

Jorgensen, S., Sigue, S.P., Zaccour, G., 2000. Dynamic cooperative advertising in a channel. J. Retail. 76 (1), 71–92.

Jorgensen, S., Taboubi, S., Zaccour, G., 2001. Cooperative advertising in a marketing channel. J. Optim. Theory Appl. 110 (1), 145–158.

Jorgensen, S., Zaccour, G., 2003. Channel coordination over time: incentive equilibria and credibility. J. Econ. Dyn. Control 27 (5), 801–822.

Jorgensen, S., Taboubi, S., Zaccour, G., 2003. Retail promotions with negative brand image effects: is cooperation possible. Eur. J. Oper. Res. 150 (2), 395–405.

Krishnan, H., Kapuscinski, R., Butz, D., 2004. Coordinating contracts for decentralized supply chains with retailer promotional effort. Manag. Sci. 50 (1), 48–63.

Khouja, M., 1999. The single-period(news-vendor)problem":literature review and suggestions for future research. Omega 27 (5), 537–553.

Lau, A.H.L., Lau, H.S., 2003. Effect of a demand-curve's shape on the optimal solutions of a multi-echelon inventory/pricing model. Eur. J. Oper. Res. 147 (3), 530–548.

Lau, A.H.L., Lau, H.S., 2001. Some two-echelon style-goods inventory models with asymmetric market information. Eur. J. Oper. Res. 134 (1), 29–42.

Lau, A.H.L., Lau, H.S., 1988. The newsboy problem with price-dependent demand distribution. IIE Trans. 20 (2), 168–175.

Li, S.X., Huang, Z., Zhu, J., Chau, Y.K.P., 2002. Cooperative advertising, game theory and manufacturer retailer supply chains. Omega 30 (5), 347–357.

Modak, N.M., Panda, S., Sana, S.S., Basu, M., 2015. Corporate social responsibility, coordination and profit distribution in a dual-channel supply chain. Pac. Sci. Rev. (online published).

Ouyang, L.Y., Ho, C.H., Su, C.H., 2008. Optimal strategy for an integrated system with variable production rate when the freight rate and trade credit are both linked to the order quantity. Int. J. Prod. Econ. 115 (1), 151–162.

Ozler, A., Tan, B., Karaesmen, F., 2009. Multi-product newsvendor problem with value-at-risk consideration. Int. J. Prod. Econ. 117, 244–255.

Panda, S., 2011. Optimal pricing and replenishment policy in a declining price sensitive environment under continuous unit cost decrease. Int. J. Math. Oper. Res. 3 (4), 431–450.

Panda, S., Saha, S., Nandi, S., 2012. Optimal pricing and economic order quantity in stock and price sensitive demand environment. Int. J. Oper. Res. 15 (4), 406–423.

Petruzzi, N.C., Dada, M., 1999. Pricing and the news vendor problem: a review with extension. Oper. Res. 47 (2), 183–194.

Qin, Y., Tang, H., Guo, C., 2007. Channel coordination and volume discounts with price-sensitive demand. Int. J. Prod. Econ. 105 (1), 43–53.

Roy, T., Chaudhuri, K.S., 2010. Optimal pricing for a perishable item under timeprice dependent demand and time-value of money. Int. J. Oper. Res. 7 (2), 133–151.

Sana, S.S., 2011. The stochastic EOQ model with random sales price. Appl. Math. Comput. 218 (2), 239–248.

Sana, S.S., 2012. Price sensitive demand with random sales price – a newsboy problem. Int. J. Syst. Sci. 43 (3), 491–498.

Silver, E.A., Pyke, D.F., Petterson, R., 1998. Inventory Management and Production Planning and Scheduling. Wiley, New York.

Soni, H., Shah, N.H., 2011. Optimal policy for fuzzy expected value production inventory model with imprecise production preparation-time. Int. J. Mach. Learn. Cybern. 2 (4), 219–224.

Taleizadeh, A.A., Niaki, S.T.A., Makui, A., 2012. Multiproduct multiple-buyer single vendor supply chain problem with stochastic demand, variable lead-time, and multi-chance constraint. Expert Syst. Appl. 39 (3), 5338–5348.

Taleizadeh, A.A., Kalantari, S.S., Cardenas-Barron, L.E., 2015. Determining optimal price, replenishment lot size and number of shipments for an EPQ model with rework and multiple shipments. J. Ind. Manag. Optim. 11 (4), 1059–1071.

Taleizadeh, A.A., Noori-daryan, M., Cardenas-Barron, L.E., 2015. Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items. Int. J. Prod. Econ. 159, 285–295.

Wang, S.D., Zhou, Y.W., Min, J., Zhong, Y.G., 2011. Coordination of cooperative advertising models in a one-manufacturer two-retailer supply chain system. Comput. Ind. Eng. 61 (4), 1053–1071.

Wang, C.X., 2010. The loss-averse newsvendor game. Int. J. Prod. Econ. 124, 448–452.

Wee, H.M., 1997. A replenishment policy for items with a price-dependent demand and a varying rate of deterioration. Prod. Plan. Control 8 (5), 494–499.

Wu, C., Petruzzi, N.C., Chhajed, D., 2007. Vertical integration with price-setting competitive newsvendors. Decis. Sci. 38, 581–610.

Ziaee, M., Bouquard, J.L., 2010. Optimal pricing and lot sizing in VMI model. Int. J. Ind. Eng. Comput. 1, 33–44.